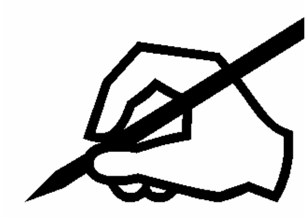
National Open School of Trinidad and Tobago



LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT Three:
Set Theory

Lesson
4

Set Operations and Venn Diagrams

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volumes 1 and 2'.

(Some helpful exercises and page numbers are given throughout the lesson e.g. Ex 1g page 19)

INTRODUCTION

In this lesson we will be looking at set operations and Venn diagrams. Many real life problems can be described in terms of sets. Therefore these problems can be solved using set theory. The Venn diagram is an important tool in solving these problems.

OBJECTIVES

At the end of this lesson you will be able to:

a) define and describe different sets;
b) determine and count the elements in the intersection and union of sets;
c) construct and use Venn diagrams to show subsets, complements, and the intersection and union of sets;
d) determine the number of elements in named subsets of not more than three intersecting sets, given the number of elements in some of the other subsets;
e) solve problems involving the use of Venn diagrams for not more than three sets;

3.1 Defining and describing sets

A set is a collection of items that share something in common, e.g. the set of vowels in the English alphabet.

Try and think of some other examples of sets.

You must be able to use phrases to describe sets. (Ex 1a 11-12 page 4)

Example: A possible description of the set P = {2, 3, 5, 7, 11, 13, 17} is P is the set of prime numbers less than eighteen.

You must also be able to list members of a set given a description of the set. (Ex 1a page 3).

Example: A = {even numbers less than 13} then A = {2, 4, 6, 8, 10, 12}.

The following sets are important.

- Finite set: a set where you can count or name all the members, e.g. the set of planets in our solar system.

- Infinite set: it is not possible to name or count all the members, e.g. the set of natural numbers.

- Null or Empty set, denoted by { } or ø: this set has no members, e.g. the set of people who have swum across the Caribbean Sea.

- Universal set, denoted U: for any given set the universal set is the set from which all its members came, e.g. a suitable universal set for B = {protractor, ruler, set square, compass, divider} is set of geometrical instruments.

- Subset: a set is a subset of another set if and only if all its members are also members of the set, e.g A = {1, 3} is a subset of B = {1, 2, 3, 4, 5}. We write $A \subset B$. Note that each set is a subset of itself and the empty set is a subset of every set.

  The number of subsets of a given set is given by S = 2^n, where n is the number of elements in the set. The number of subset that can be formed from the set B is $2^5 = 32$.

  Try and list them, remember to include the empty set and the set itself. A few has been done for you: {1}, {2}, {3}, {4}, {5}, {1,2}, {1,3},….

- Equal sets: these are sets with the same elements, e.g. A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4, 5} then A = B.

- Equivalent sets: these sets have the same number of elements, e.g. P = {a, b, c, d, e} and B = {1, 2, 3, 4, 5} then P and B are equivalent.

ACTIVITY 1

You can get valuable practice by doing a few items from Ex 1a to Ex 1e, page 3 to 6.

3.2 Set Builder Notation

You may use set builder notation to describe a set. Consider the following notation: $\{x$ **: …**$\}$. This means 'the set of all x such that'.

Example: Describe the following set using set builder notation.

A = {1, 2, 3, 4, 5}.

Solution: A = $\{x : 0 < x \leq 5, x \in W\}$, i.e. the set of all x such that x is greater than 0 but less than or equal to 5 and x is a whole number.

ACTIVITY 2

Describe the following set using set builder notation.

X = {−3, −2, −1, 0, 1, 2, 3, 4}

3.3 Set Operations and Venn Diagrams

A Venn diagram consists of a rectangle that represents the universal set and circles that represent the subsets. The following are some very important subsets represented in Venn diagrams. The following sets will be used to develop the examples throughout.

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

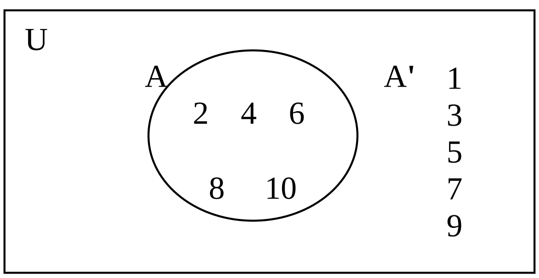
A = { 2, 4, 6, 8, 10}

B = {3, 6, 9}

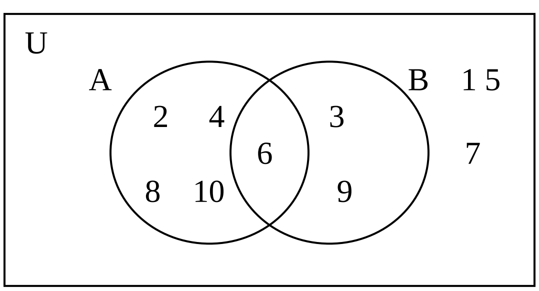
C = {2, 4}

D = {1, 3, 5, 7, 9}

Complement: the complement of a set A, written as A' (A prime) is the set of members (or elements) in the universal set but not in A.

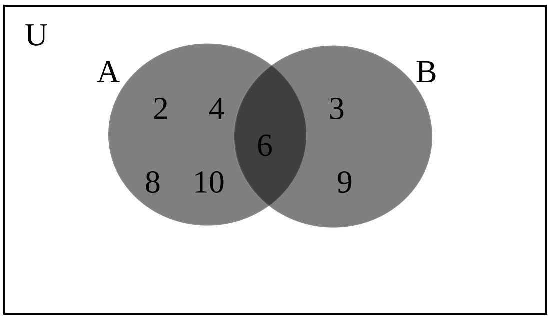


Intersection of two sets: This is a set that contains all the members that belong to both sets, i.e. A∩B = $\{x : x \in A \text{ and } x \in B\}$. The symbol ∩ is used for intersection and ∈ means 'a member of'.

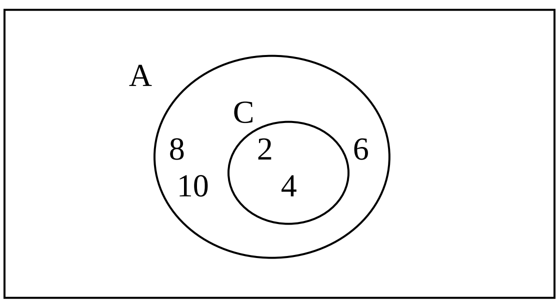


Clearly A∩B = { 6 }.

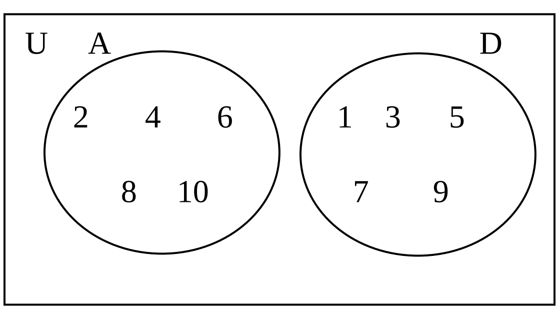
Can you identify the set B∩A'?
Union of two sets: This is the set of all elements that are in either of the two sets, i.e AUB = $\{x : x \in A \text{ or } x \in B\}$. Therefore AUB = $\{2, 3, 4, 6, 8, 9, 10\}$. This is represented by the shaded area in the following Venn diagram. Note U is used for union.



Subset: C⊂A



Disjoint Sets: Two sets are disjointed if they have no common members.



ACTIVITY 3

(Ex 1g page 19)

3.4 Number of elements in a given set

If A = { 2, 4, 6, 8, 10} then the number of elements in A or n(A) = 5.

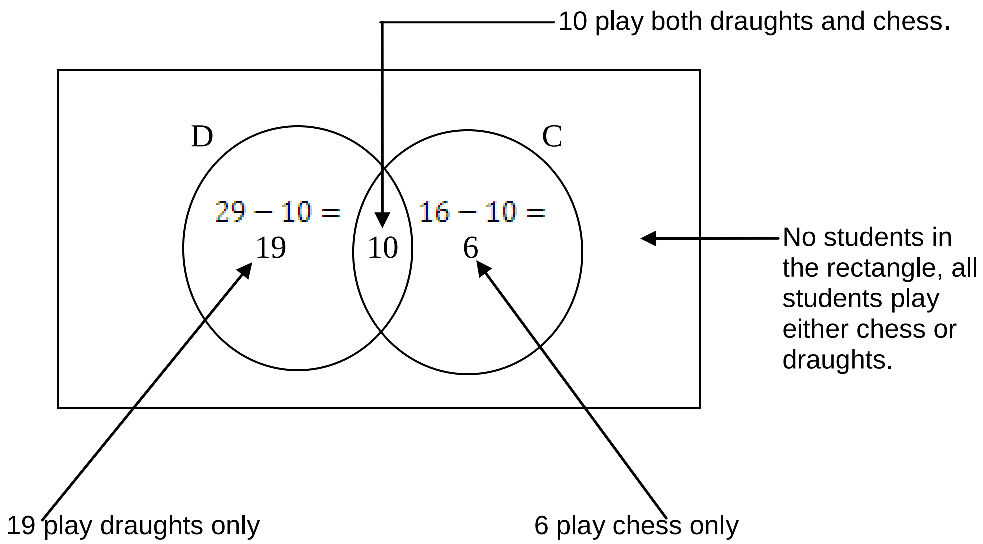
3.5 Solving problems using Venn diagrams

You can use Venn diagrams to solve problems that involve finding the number of elements is certain subsets of two intersecting sets.

Example: In a class of 35 students, 29 play draughts, 16 play chess and 10 play both draughts and chess. Each student plays either draughts or chess. Find the number of students who play:

(a) Draughts only (b) chess only

Solution:



You may use the following formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ to solve simple numerical problems.

Example: In a group of 60 students, 31 speak French, 23 speak Spanish and 14 speak neither French nor Spanish. Determine the number of students who speak both French and Spanish.

Solution: $n(F \cup S) = 60 - 14 = 46$;

$n(F) = 31;$

$n(S) = 23;$

$46 = 31 + 23 - n(F \cap S)$

$n(F \cap S) = 31 + 23 - 46$

$= 54 - 46 = 8.$

8 students speak both French and Spanish.

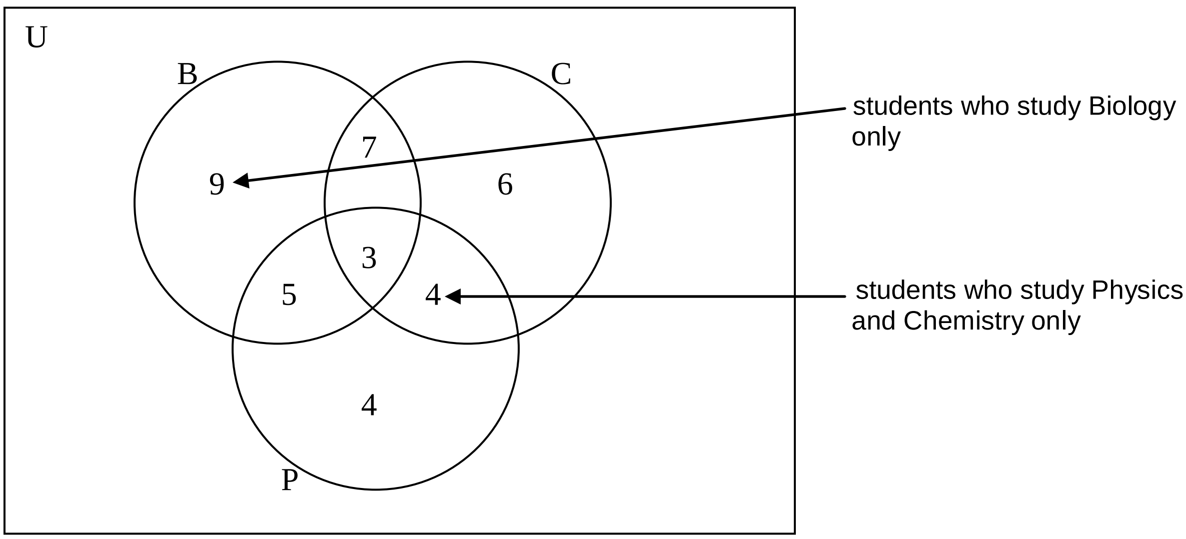
Let us look at a problem involving three sets.

There are 38 students in form 5. All students study science.
16 study Physics,
20 study Chemistry,
24 study Biology,
7 study Physic and Chemistry,
8 study Physics and Biology,
10 study Biology and Chemistry,
3 study all three subjects.

(a) Draw a carefully labelled Venn diagram to represent the data.
(b) Determine the number of students who study at least two subjects.
(c) Evaluate the number of students who study Physics and Chemistry only.
(d) State the number of students who study Biology only.

Solution: when drawing a Venn diagram you need not put arrows, they are included here to assist you.

a)



b) The number of students who study at least two subject are those students in the intersections of the circle.

7 + 5 + 3 + 4 = 19 students.

c) 4 students studied Physics and Chemistry only.

d) 9 students study biology only.

ASSESSMENT

Ex 1h page 13
Ex 12a page 716 vol. 2

CONCLUSION

We have used sets and Venn diagrams to solve simple numerical problems. The next unit deals with Consumer Arithmetic. Many computational skills that are used daily will be developed.